group. We see as in the preceding paragraph that H cannot contain any selfconjugate subgroup unless this subgroup is a regular group of order $p+1$. As this regular subgroup has to contain p subgroups that are conjugate in H it cannot involve any substitution whose order exceeds 2. Hence we have the

THEOREM. *If $p+1$ is not a power of 2 then the substitutions of order p (p being any odd prime number) that are contained in a group of degree $p+1$ generate a simple group. If this simple group does not coincide with the entire group it is selfconjugate and the corresponding quotient group is a cyclical and its order is a divisor of $p-1$.* If $p+1$ is a power of 2, the group generated by the substitutions of order p that are contained in a group of degree $p+1$ cannot contain any selfconjugate subgroup except perhaps the regular group of order $p+1$ which contains no substitution whose order exceeds 2.*

Cornell University, April 3, 1899.

*From this theorem it follows directly that the three primitive groups of degree 12 and orders 660, 7920, and 95040, respectively, are simple. The last of these three groups is the well known five-fold transitive group of Mathien.

ON SYMMETRIC FUNCTIONS.

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[Continued from March Number.]

B. FUNDAMENTAL RELATIONS FOR SYMMETRIC FUNCTIONS.

1. FUNDAMENTAL RELATIONS BETWEEN COEFFICIENTS.

(1). *Derivation of the relations.*

In A, 4, (3) we have already obtained one of the relations, viz :

$$\begin{pmatrix} 0^{\lambda_0}1^{\lambda_1}\ldots\ldots\ldots\ldots n^{\lambda_n} \\ b_0{}^m\beta_1{}^{\kappa_1}\beta_2{}^{\kappa_2}\ldots\ldots\beta_m{}^{\kappa_n}\end{pmatrix}=(-1)^{mn}\begin{pmatrix}(m-\varkappa_1)(m-\varkappa_2)\ldots\ldots(m-\varkappa_n) \\ a_0{}^n(\alpha\lambda_0)^n(\alpha\lambda_1)^{n-1}\ldots\ldots(\alpha\lambda_n)^0\end{pmatrix}.$$

If in the equations $b_0x^n+b_1x^{n-1}+\ldots\ldots+b_n'=0$, and $a_0x^m+a_1x^{m-1}+\ldots\ldots+a_m=0$ (cf. loc. cit.), we substitute $x=1/y$, b_r becomes b_{n-r}, a_r becomes a_{m-r}, and $b_0{}^m\Sigma\beta_1{}^{\kappa_1}\beta_2{}^{\kappa_2}\ldots\ldots\beta_n{}^{\kappa_n}$ becomes

$$b_n{}^m\,\Sigma\frac{1}{\beta_1{}^{\kappa_1}\beta_2{}^{\kappa_2}\ldots\ldots\beta_n{}^{\kappa_n}}=\frac{b_n{}^m}{(\beta_1\beta_2\ldots\ldots\beta_n)^m}\Sigma\beta_1{}^{m-\kappa_1}\beta_2{}^{m-\kappa_2}\ldots\ldots\beta_n{}^{m-\kappa_n}$$

$$=(-1)^{mn}b_0{}^m\Sigma\beta_1{}^{m-\kappa_1}\beta_2{}^{m-\kappa_2}\ldots\ldots\beta_n{}^{m-\kappa_n}.$$

Similarly $a_0{}^n\Sigma(\alpha\lambda_0)^n(\alpha\lambda_1)^{n-1}\ldots\ldots(\alpha\lambda_n)^0$ becomes $(-1)^{mn}a_0{}^n\Sigma(\alpha\lambda_n)^n(\alpha\lambda_{n-1})^{n-1}\ldots\ldots(\alpha\lambda_0)^0$. We have therefore,

$$\begin{pmatrix} 0^{\lambda_0}1^{\lambda_1}\dots\dots\dots\dots n^{\lambda_n} \\ b_0{}^m\beta_1{}^{\kappa_1}\beta_2{}^{\kappa_2}\dots\beta_n{}^{\kappa_n}\end{pmatrix}=(-1)^{mn}\begin{pmatrix} 0^{\lambda_n}1^{\lambda_{n-1}}\dots\dots\dots\dots n^{\lambda_0} \\ b_0{}^m\beta_1{}^{m-\kappa_1}\beta_2{}^{m-\kappa_2}\dots\beta_n{}^{m-\kappa_n}\end{pmatrix} \text{ and}$$

$$\begin{pmatrix}(m-\varkappa_1)(m-\varkappa_2)\dots(m-\varkappa_n) \\ a_0{}^n(\alpha\lambda_0)^n(\alpha\lambda_1)^{n-1}\dots(\alpha\lambda_n)^0\end{pmatrix}=(-1)^{mn}\begin{pmatrix}\varkappa_1\varkappa_2\dots\dots\dots\dots\varkappa_n \\ a_0{}^n(\alpha\lambda_n)^n(\alpha\lambda_{n-1})^{n-1}\dots(\alpha\lambda_0)^0\end{pmatrix}$$

The last relations correspond to reciprocal terms in the resultant theory. [Terms of the resultant like $(a_{r_1})^{p_1}(a_{r_2})^{p_2}\dots(a_{r_\mu})^{p_\mu}(b_{s_1})^{q_1}(b_{s_2})^{q_2}\dots(b_{s_\nu})^{q_\nu}$ and $(a_{m-r_\mu})^{p_\mu}(a_{m-r_{\mu-1}})^{x_{\mu-1}}\dots(a_{m-r_1})^{p_1}(b_{n-s_\nu})^{q_\nu}(b_{n-s_{\nu-1}})^{q_{\nu-1}}\dots(b_{n-s_1})^{q_1}$ are called reciprocal terms.] Since one member in each is equal numerically to the terms of the relation already obtained, the four terms are numerically equal. Taken together we shall call them the fundamental relations for the coefficients of symmetric functions.

(2). *Other notation for the fundamental relation.*

We will now re-write the relations given in (1), replacing such expressions as $b_0{}^m\beta_1{}^{\kappa_1}\beta_2{}^{\kappa_2}\dots\beta_n{}^{\kappa_n}$ and $a_0{}^n(\alpha\lambda_0)^n(\alpha\lambda_1)^{n-1}\dots(\alpha\lambda_n)^0$ by $0^m\varkappa_1\varkappa_2\dots\varkappa_n$ and $0^n n^{\lambda_0}(n-1)^{\lambda_1}\dots 0^{\lambda_n}$, respectively, $(n-r)^{\lambda_r}$, *e. g.*, signifying, as before $(\alpha\lambda_r)^{n-r}$ signified, that λ_r roots have the exponent $(n-r)$. We may then write the fundamental relations :

$$\begin{pmatrix}0^{\lambda_0}1^{\lambda_1}\dots\dots n^{\lambda_n} \\ 0^m\varkappa_1\varkappa_2\dots\varkappa_m\end{pmatrix}=\begin{pmatrix}\varkappa_1\varkappa_2\dots\dots\varkappa_n \\ 0^n0^{\lambda_0}1^{\lambda_1}\dots n^{\lambda_n}\end{pmatrix}=(-1)^{mn}\begin{pmatrix}0^{\lambda_n}1^{\lambda_{n-1}}\dots\dots\dots n^{\lambda_0} \\ 0^m(m-\varkappa_1)(m-\varkappa_2)\dots(m-\varkappa_n)\end{pmatrix}$$

$$=(-1)^{mn}\begin{pmatrix}(m-\varkappa_1)(m-\varkappa_2)\dots(m-\varkappa_n) \\ 0^n0^{\lambda_n}1^{\lambda_{n-1}}\dots\dots\dots n^{\lambda_0}\end{pmatrix}.^*$$

2. Fundamental and Necessary Conditions of Coefficients.

We will next consider some relations of conditions of the coefficients themselves.

(1). *First condition.*

Since $b_0{}^m\Sigma\beta_1{}^{\kappa_1}\beta_2{}^{\kappa_2}\dots\beta_n{}^{\kappa_n}=b_0{}^m(\beta_1\beta_2\dots\beta_n)^{\kappa_n}\Sigma\beta_1{}^{\kappa_1-\kappa_n}\beta_2{}^{\kappa_2-\kappa_n}\dots(\beta_{n-1})^{\kappa_{n-1}-\kappa_n}=(-1)^{n\kappa_n}b_0{}^{m-\kappa_n}b_n{}^{\kappa_n}\Sigma\beta_1{}^{\kappa_1-\kappa_n}\beta_2{}^{\kappa_2-\kappa_n}\dots(\beta_{n-1})^{\kappa_{n-1}-\kappa_n}$, supposing $\varkappa_1, \varkappa_2, \dots\varkappa_n$ to be in order of descending magnitude, it follows that, at least, either $\lambda_n>0$, (and $\lambda_n\geq\varkappa_n$), or else $\lambda_n=0$, and $\varkappa_n=0$. This corresponds to the condition in the resultant $R_{m,n}$, that, at least, one of the two factors b_n or a_m must be present in every term.

(2). *Second condition.*

Again, since by 1, (2),

*It might seem as if the distinction of a and b is lost by this notation. A little reflection shows that this distinction does not need to be kept. In fact the theory is clearer without it.

$$\begin{pmatrix} 0^{\lambda_0} 1^{\lambda_1} \ldots\ldots n^{\kappa_n} \\ 0^m \varkappa_1 \varkappa_2 \ \ldots . \varkappa_n \end{pmatrix} = (-1)^{mn} \begin{pmatrix} 0^{\lambda_n} 1^{\lambda_{n-1}} \ldots\ldots\ldots\ldots\ldots\ldots n^{\lambda_0} \\ 0^m (m-\varkappa_1)(m-\varkappa_2) \ldots . (m-\varkappa_n) \end{pmatrix}$$

it follows by the previous proof, that, at least, either $\lambda_0>0$, (and $\lambda_0 \geqq m-\varkappa_1$), or else $\lambda_0=0$, $m-\varkappa_1=0$, *i. e.*, $\varkappa_1=m$. This corresponds to the condition in the resultant $R_{m,n}$, that, at least of the two factors b_0 and a_0, one must be present in every term.

When these conditions are not satisfied, the coefficient is equal to zero. It will be shown later that if $\lambda_n=\lambda_{n-1}=\ldots .=\lambda_r=0$. then $\varkappa_n=\varkappa_{n-1}=\ldots .=0$, and if $\lambda_0=\lambda_1=\ldots .=\lambda_r=0$, then $m-\varkappa_1=m-\varkappa_2=\ldots .=m-\varkappa_r=0$, *i. e.*, $\varkappa_1=\varkappa_r=\ldots .=\varkappa_r=m$; otherwise the coefficient is zero (D, 3 and 4).

3. Fundamental Equations of Condition Connecting the Subscripts and Exponents.

From the theorems of order and weight of symmetric functions, and from the relations between the four coefficients given in 1, (2), we have the following equations of condition connecting the subscripts and exponents :

(1). $\lambda_0+\lambda_1=\ldots .\lambda_n=m$,

(2). $\lambda_1+2\lambda_2+\ldots .n\lambda_n=\varkappa_1+\varkappa_2+\ldots .\varkappa_n$,

(3). $\lambda_{n-1}+2\lambda_{n-2}+\ldots .n\lambda_0=mn-(\varkappa_1+\varkappa_2+\ldots .\varkappa_n)$, and by adding mn to both sides of (2) that equation may be written,

(4). $\lambda_1+2\lambda_2+\ldots .n\lambda_n+mn-(\varkappa_1+\varkappa_2+\ldots .\varkappa_n)=mn$, the same equations which we should have in the theory of the resultant, excepting (3) which can be derived from (1) by multiplying that equation by n, and then substituting the value of $\varkappa_1+\varkappa_2+\ \ldots \varkappa_n$ from (2).

C. NORMAL FORMS AND REDUCIBLE FORMS.

As in the theory of the resultant so also here we have normal forms and reducible forms. A normal form will be characterized by having $\lambda_0>0$, $\lambda_n>0$, $\varkappa_1=m$, $\varkappa_n=0$. All other forms, the completely reducible forms excepted, may be reduced to such as are normal forms having a lower m or n. As in the rerultant theory, so here there are four kinds of reduction, with the four corresponding kinds of derivation. In the next divisions we will briefly treat of these.

D. REDUCTION.

1. Reduction in the Case where $\varkappa_1 \neq m$.

In this case λ_0 must be greater than zero [cf. B, 2, (2)]. It is evident that we may divide by the factor b_0 in the given term as well as in $b_0{}^m \Sigma \beta_1{}^{\kappa_1} \beta_2{}^{\kappa_2} \ldots . \beta_n{}^{\kappa_n}$ without affecting the value of the coefficient of the term, and that therefore,

$$\begin{pmatrix} 0^{\lambda_0} 1^{\lambda_1} \ \ldots . \ n^{\lambda_n} \\ 0^m \varkappa_1 \varkappa_2 \ \ldots\ldots \varkappa_n \end{pmatrix} = \begin{pmatrix} 0^{\lambda_0-1} 1^{\lambda_1} \ \ldots\ldots n^{\lambda_n} \\ 0^{m-1} \varkappa_1 \varkappa_2 \ \ldots\ldots \varkappa_n \end{pmatrix},$$

and this reduction may be continued until the exponent of b_0 below is equal to $\varkappa_1$. We then have

$$\begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots n^{\lambda_n} \\ 0^m \varkappa_1 \varkappa_2 \ldots\ldots \varkappa_n \end{pmatrix} = \begin{pmatrix} 0^{\lambda_0-\lambda} 1^{\lambda_1} \ldots\ldots n^{\lambda_n} \\ 0^{\kappa_1} \varkappa_1 \varkappa_2 \ldots\ldots \varkappa_n \end{pmatrix},$$

where $m-\lambda=\varkappa$, $\lambda_0-\lambda\geqq 0$.

This reduction corresponds to the third kind of reduction in the theory of the resultant.

2. Reduction in the Case where $\varkappa_n \neq 0$.

Here we must have $\lambda_n>0$. [Cf. B, 2, (1)], and the work already done in B, 2, shows that

$$\begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots n^{\lambda_n} \\ 0^m \varkappa_1 \varkappa_2 \ldots\ldots \varkappa_n \end{pmatrix} = (-1)^{n\kappa_n} \begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots\ldots\ldots\ldots\ldots n^{\lambda_n-\kappa_n} \\ 0^{m-\kappa_n} (\varkappa_1-\varkappa_n)(\varkappa_2-\varkappa_n) \ldots (\varkappa_{n-1}-\varkappa_n) \end{pmatrix}$$

or we may prove it otherwise as follows: By B, 1, (2),

$$=\begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots n^{\lambda_n} \\ 0^m \varkappa_1 \varkappa_2 \ldots\ldots \varkappa_n \end{pmatrix} = (-1)^{mn} \begin{pmatrix} 0^{\lambda_n} 1^{\lambda_{n-1}} \ldots\ldots\ldots\ldots n^{\lambda_0} \\ 0^m (m-\varkappa_1)(m-\varkappa_2) \ldots (m-\varkappa_n) \end{pmatrix}.$$

By 1, the right member of this can be reduced. We get

$$\begin{pmatrix} 0^{\lambda_n} 1^{\lambda_{n-1}} \ldots\ldots\ldots\ldots n^{\lambda_0} \\ 0^m (m-\varkappa_n)(m-\varkappa_{n-1}) \ldots (m-\varkappa_1) \end{pmatrix} = \begin{pmatrix} 0^{\lambda_n-\kappa_n} 1^{\lambda_{n-1}} \ldots\ldots\ldots\ldots n^{\lambda_0} \\ 0^{m-\kappa_n} (m-\varkappa_n)(m-\varkappa_{n-1}) \ldots (m-\varkappa_1) \end{pmatrix}.$$

Again by B, 1, (2), $\begin{pmatrix} 0^{\lambda_n-\kappa_n} 1^{\lambda_{n-1}} \ldots\ldots\ldots\ldots n^{\lambda_0} \\ 0^{m-\kappa_n} (m-\varkappa_n)(m-\varkappa_{n-1}) \ldots (m-\varkappa_1) \end{pmatrix}$

$$=(-1)^{(m-\kappa_n)n} \begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots\ldots\ldots\ldots\ldots n^{\lambda_n-\kappa_n} \\ 0^{m-\kappa_n} (\varkappa_1-\varkappa_n)(\varkappa_2-\varkappa_n) \ldots (\varkappa_{n-1}-\varkappa_n) \end{pmatrix}.$$

Substituting, we get

$$\begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots n^{\lambda_n} \\ 0^m \varkappa_1 \varkappa_2 \ldots\ldots \varkappa_n \end{pmatrix} = (-1)^{n\kappa_n} \begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots\ldots\ldots\ldots\ldots n^{\lambda_n-\kappa_n} \\ 0^{m-\kappa_n} (\varkappa_1-\varkappa_n)(\varkappa_2-\varkappa_n) \ldots (\varkappa_{n-1}-\varkappa_n) \end{pmatrix}$$

as before. This corresponds to the first kind of reduction in the resultant theory.

3. Reduction in the Case where $\lambda_n=0$.

We must have $\kappa_n=0$ in this case. By B, 1, (2) we have

$$\begin{pmatrix} 0^{\lambda_0}\, 1^{\lambda_1} \ldots\ldots n^{\lambda_n} \\ 0^m \varkappa_1 \varkappa_2 \ldots \varkappa_{n-1} 0 \end{pmatrix} = \begin{pmatrix} 0 \;\; \varkappa_{n-1} \;\; \varkappa_{n-2} \ldots \varkappa_1 \\ 0^n \;\; 0^{\lambda_0} \;\; 1^{\lambda_1} \ldots\ldots n^0 \end{pmatrix}.$$

By D, 1, the right member can be reduced, and

$$\begin{pmatrix} 0 \;\; \varkappa_{n-1} \;\; \varkappa_{n-2} \ldots\ldots\ldots \varkappa_1 \\ 0^n \;\; 0^{\lambda_0} \;\; 1^{\lambda_1} \ldots (n-1)^{\lambda_{n-1}} \;\; n^0 \end{pmatrix} = \begin{pmatrix} 0 \;\; \varkappa_{n-1} \;\; \varkappa_{n-2} \ldots\ldots\ldots \varkappa_1 \\ 0^n \;\; 0^{\lambda_0} \;\; 1^{\lambda_1} \ldots (n-1)^{\lambda_{n-1}} \end{pmatrix}$$

$$=\begin{pmatrix} \varkappa_{n-1}\,\varkappa_{n-2}\ldots\ldots\ldots\ldots\varkappa_1 \\ 0^{n-1}\,0^{\lambda_0}\,1^{\lambda_1}\ldots(n-1)^{\lambda_{n-1}} \end{pmatrix}.$$

Again by B, 1, (2),

$$\begin{pmatrix} \varkappa_{n-1}\,\varkappa_{n-2}\ldots\ldots\ldots\ldots\varkappa_1 \\ 0^{n-1}0^{\lambda_0}\,1^{\lambda_1}\ldots(n-1)^{\lambda_{n-1}} \end{pmatrix}=\begin{pmatrix} 0^{\lambda_0}\,1^{\lambda_1}\ldots(n-1)^{\lambda_{n-1}} \\ 0^m\varkappa_1\varkappa_2\ldots\ldots\varkappa_{n-1} \end{pmatrix},$$

and by substituting,

$$\begin{pmatrix} 0^{\lambda_0}\,1^{\lambda_1}\ldots(n-1)^{\lambda_{n-1}}n^0 \\ 0^m\varkappa_1\varkappa_2\ldots\ldots\varkappa_{n-1}\,0 \end{pmatrix}=\begin{pmatrix} 0^{\lambda_0}\,1^{\lambda_1}\ldots(n-1)^{\lambda_{n-1}} \\ 0^m\varkappa_1\varkappa_2\ldots\ldots\varkappa_{n-1} \end{pmatrix}.$$

In a similar way it will follow that if $\lambda_n=\lambda_{n-1}=\ldots=\lambda_r=0$, $\varkappa_n=\varkappa_{n-1}=\ldots=\varkappa_r=0$, and

$$\begin{pmatrix} 0^{\lambda_0}\,1^{\lambda_1}\ldots r^0(r+1)^0\ldots n^0 \\ 0^m\varkappa_1\varkappa_2\ldots\ldots\varkappa_{r-1}0^{n-r+1} \end{pmatrix}=\begin{pmatrix} 0^{\lambda_0}\,1^{\lambda_1}\ldots(r-1)^{\lambda_{r-1}} \\ 0^m\varkappa_1\varkappa_2\ldots\ldots\varkappa_{r-1} \end{pmatrix}.$$

The meaning of this last formula is that a function of the roots of an equation of the nth degree, which involves only $(r-1)$ roots at a time, has the same coefficient of the term involving $b_0^{\lambda_0}b_1^{\lambda_1}\ldots(b_{r-1})^{\lambda_{r-1}}$, as the same function of the roots of an equation of the $(r-1)$st degree which involves all the roots; and the reduction corresponds to the reduction of the second kind in the resultant theory.

4. Reduction in the Case where $\lambda_0=0$.

We must have here $\varkappa_1=m$. By B, 1, (2), and D, 3,

$$\begin{pmatrix} 1^{\lambda_1}2^{\lambda_2}\ldots n^{\lambda_n} \\ 0^{\kappa_1}\varkappa_1\varkappa_2\ldots\varkappa_n \end{pmatrix}=(-1)^{\kappa_1 n}\begin{pmatrix} 0^{\lambda_n}1^{\lambda_{n-1}}\ldots\ldots\ldots(n-1)^{\lambda_1} \\ 0^{\kappa_1}(\varkappa_1-\varkappa_n)(\varkappa_1-\varkappa_{n-1})\ldots(\varkappa_1-\varkappa_2) \end{pmatrix}.$$

Again by B, 1, (2),

$$\begin{pmatrix} 0^{\lambda_n}1^{\lambda_{n-1}}\ldots\ldots\ldots(n-1)^{\lambda_1} \\ 0^{\kappa_1}(\varkappa_1-\varkappa_n)(\varkappa_1-\varkappa_{n-1})\ldots(\varkappa_1-\varkappa_2) \end{pmatrix}=(-1)^{\kappa_1(n-1)}\begin{pmatrix} 0^{\lambda_1}\,1^{\lambda_2}\ldots(n-1)^{\lambda_n} \\ 0^{\kappa_1}\varkappa_2\varkappa_3\ldots\ldots\varkappa_n \end{pmatrix}.$$

Therefore $\begin{pmatrix} 1^{\lambda_1}\,2^{\lambda_2}\ldots\ldots n^{\lambda_n} \\ 0^{\kappa_1}\varkappa_1\varkappa_2\ldots\varkappa_n \end{pmatrix}=(-1)^{\kappa_1}\begin{pmatrix} 0^{\lambda_1}\,1^{\lambda_2}\ldots(n-1)^{\lambda_n} \\ 0^{\kappa_1}\varkappa_2\varkappa_3\ldots\ldots\varkappa_n \end{pmatrix}.$

If $\lambda_0=\lambda_1=\ldots=\lambda_{r-1}=0$.

$$\begin{pmatrix} r^{\lambda_r}(r+1)^{\lambda_{r+1}}\ldots n^{\lambda_r} \\ 0^m\,\varkappa_1\,\varkappa_2\ldots\ldots\ldots\varkappa_n \end{pmatrix}=(-1)^{mn}\begin{pmatrix} 0^{\lambda_n}\,1^{\lambda_{n-1}}\ldots\ldots\ldots(n-r)^{\lambda_r} \\ 0^m(m-\varkappa_1)(m-\varkappa_2)\ldots(m-\varkappa_n) \end{pmatrix},$$

and the right member of this is either zero, or else

$$m-\varkappa_1=m-\varkappa_2=\ldots=m-\varkappa_r=0,\ \varkappa_1=\varkappa_2=\ldots=\varkappa_r=m,$$

and the reduction of this section gives

$$\begin{pmatrix} r^{\lambda_r}(r+1)^{\lambda_{r+1}}\ldots n^{\lambda_n} \\ 0^m\varkappa_1\,\varkappa_2\ldots\ldots\ldots\varkappa_n \end{pmatrix}=(-1)^{\kappa_1 r}\begin{pmatrix} 0^{\lambda_r}\,1^{\lambda_{r+1}}\ldots(n-r)^{\lambda_n} \\ 0^{\kappa_1}\varkappa_{r+1}\,\varkappa_{r+2}\ldots\ldots\varkappa_n \end{pmatrix}.$$

This kind of reduction corresponds to reduction of the fourth kind in the resultant theory.

[To be continued.]